

## Row Echelon Form (REF)

A matrix is in row echelon form if and only if

- the first (leftmost) non-zero entry in each row is 1 (called the leading 1),
- the leading 1 in each row (except row 1) is to the right of the leading 1 in the row above it,
- and all rows which contain only 0 are below all rows which contain any non-zero entry.

Are these matrices in REF ? If not, why not ?

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 0 & -1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 1 & 4 & -3 & -2 \\ 0 & 0 & 1 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

## Reduced Row Echelon Form (RREF)

A matrix is in reduced row echelon form if and only if

- it is in row echelon form,
- and all columns which contain a leading 1 contain only 0 in all other entries.

Are these matrices in REF ? If not, why not ?

$$\begin{bmatrix} 1 & 0 & -1 & -2 & 4 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -3 & 0 & 4 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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## Gaussian Elimination Pivot Method

- Step 1: Find the first (leftmost) column which contains a non-zero entry
- Step 2: Choose a pivot in that column (to be used to replace all lower entries in that column with 0)
- Step 3: SWAP to move the pivot's row to the top
- Step 4: SCALE to turn the pivot into 1
- Step 5: REPLACE each row below the pivot's row  
by adding the multiple of the pivot's row which gives a 0 under the pivot
- Step 6: Cover up the pivot's row & repeat the entire process (stop when matrix is in row echelon form)

## Gauss-Jordan Elimination (after matrix is in row echelon form)

- Step 7: Find the last (rightmost) column which contains a pivot (leading 1)
- Step 8: REPLACE each row above the pivot's row  
by adding the multiple of the pivot's row which gives a 0 above the pivot
- Step 9: Cover up the pivot's row & repeat the entire process (stop when matrix is in reduced row echelon form)

**The following examples should not require fractions if solved using the processes above.**

Example 1:

$$\begin{aligned} 3x + 2y - z &= -1 \\ 5x + y - 3z &= -2 \\ 2x + 4y + 2z &= 2 \end{aligned}$$

Example 2:

$$\begin{aligned} 4x + 6y - 3z &= -5 \\ 3x + 4y + z &= 11 \\ -x - 2y + z &= 1 \end{aligned}$$

Example 3:

$$\begin{aligned} 3x + 4y - 11z &= -17 \\ 2x + y - 4z &= 5 \\ -x - 2y + 5z &= -9 \end{aligned}$$

Example 4:

$$\begin{aligned} 3x + 5y - 9z &= 14 \\ 2x - 3y + 13z &= 3 \\ -x + 2y - 8z &= -1 \end{aligned}$$

Example 5:

$$\begin{aligned} 2x + 4y + 11z &= 10 \\ x + 2y + 7z &= 5 \\ 3x + 4y + 9z &= 13 \end{aligned}$$